Hydraulic Control and Capture Zones:

Principle of Superposition:

When governing equations consist of LINEAR differential equations, the effect on hydraulic head of several wells pumping simultaneously is ADDITIVE.

- Are the following 2-D governing equations LINEAR?

Confined Aquifer:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} K_x \frac{\partial h}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial h}{\partial y}$$

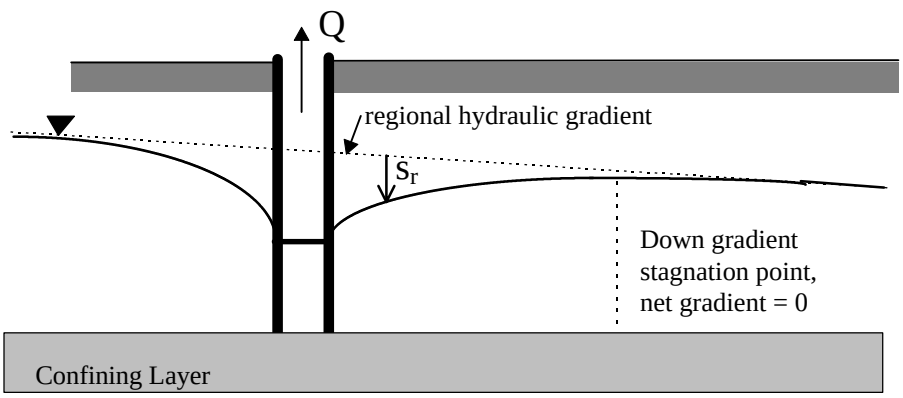
Unconfined Aquifer:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} K_x h \frac{\partial h}{\partial x} + \frac{\partial}{\partial y} K_y h \frac{\partial h}{\partial y}$$

If $s_w << h_0$ (draw down is small compared to saturated thickness), we may use superpositioning for unconfined aquifers. Note that actual drawdown will be *greater* than that predicted by superpositioning.

Capture Zones:

- Superposition well drawdowns on hydraulic gradient.



Create final groundwater contours by subtracting drawdowns from regional gradient.

Example 1. Capture zone for a single well in a uniform flow field using superpositioning.

Assume: h_0 = 40 m
h_w = 38 m
regional gradient, dh/dx = -0.01
$K = 10^{-4}$ m/s
$2r_w$ = 0.2 m

Also: 2-D flow (no vertical gradients) in a water table aquifer
Incompresible fluid/nondeformable media
Homogeneous, isotropic aquifer
Steady state conditions

Example 1. Capture zone for a single well in a uniform flow field using superpositioning (continued).

Step 1: Determine head contours for pumping well in *x,y* plane *[$h_{pw}(x,y)$]:*

Can use analytical equation: $$h^2 - h_w^{\ 2} = \frac{\left(h_0^{\ 2} - h_w^{\ 2}\right)}{\ln\left(R/r_w\right)} \ln\left(r/r_w\right)$$

Need estimate of radius of influence, *R*:

$R \approx 3000\,(h_0 - h_w)\,K^{0.5} = 3000(2)(10^{-4})^{0.5} = 60\ m$

Plug in values for h_0, h_w, R, r_w; solve for $h_{pw}(r)$:

$$h_{pw}(r) = \sqrt{24.387(\ln r) + 1500.15}$$

Can solve here, or convert to *x, y* coordinates: $r = (x^2 + y^2)^{0.5}$

$$h_{pw}(x,y) = \sqrt{24.387 \ln\{[(x-a)^2 + (y-b)^2]^{0.5}\} + 1500.15}$$

where well is centered at coordinates *(a,b)*.

Convert to drawdown from h_0: $s_{pw}(x,y) = h_{pw}(x,y) - h_0$

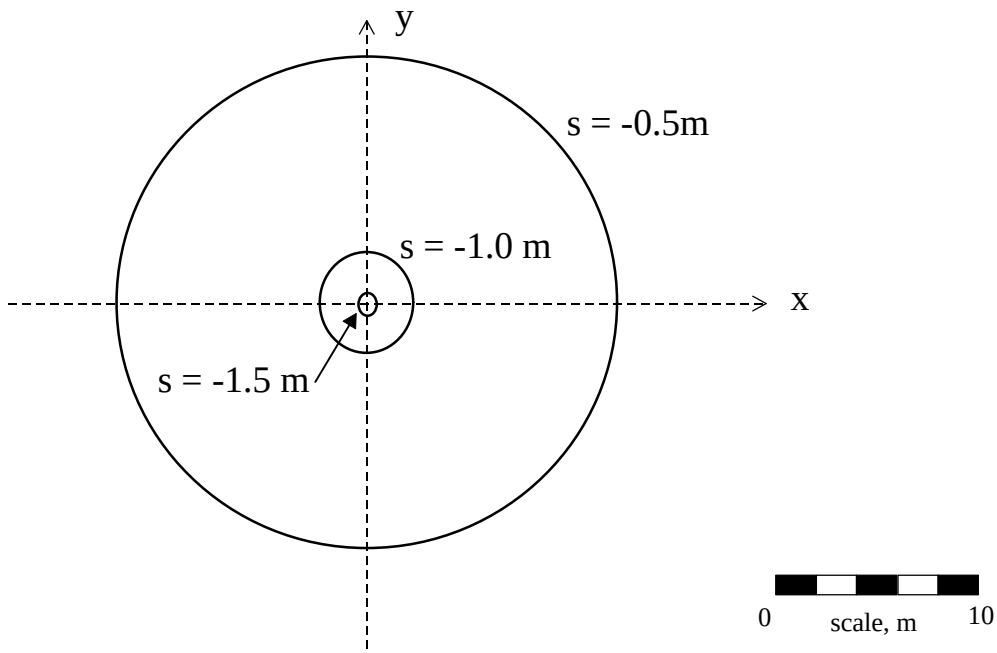
Note sign convention: $s < 0 \rightarrow h < h_0$

$s > 0 \rightarrow h > h_0$

See spreadsheet table for $s_{pw}(x,y)$

Example 1. Capture zone for a single well in a uniform flow field using superpositioning (continued).

Plot of $s_{pw}(x,y)$ with well centered at $(x,y) = (0,0)$:



Step 2: Determine head from regional gradient in x, y plane (without effects of pumping):

$$\int_{h_0}^{h} dh = -0.01 \int_{a}^{x} dx$$

$$h_{rg}(x,y) = -0.01(x-a) + h_0$$

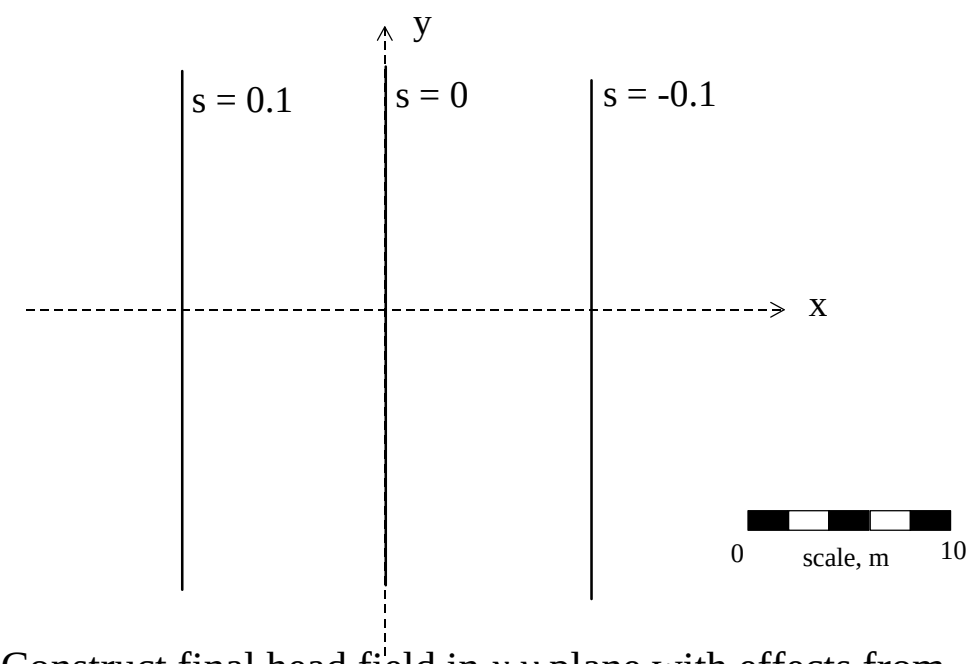
where $h = h_0$ at $x=a$.

Example 1. Capture zone for a single well in a uniform flow field using superpositioning (continued).

Convert to drawdown from h_0: $s_{rg}(x,y) = h_{rg}(x,y) - h_0$

See spreadsheet table for $s_{rg}(x,y)$

Plot of $s_{rg}(x,y)$ with $h = h_0$ at $x = 0$:



Step 3: Construct final head field in *x,y* plane with effects from both regional gradient and pumping.

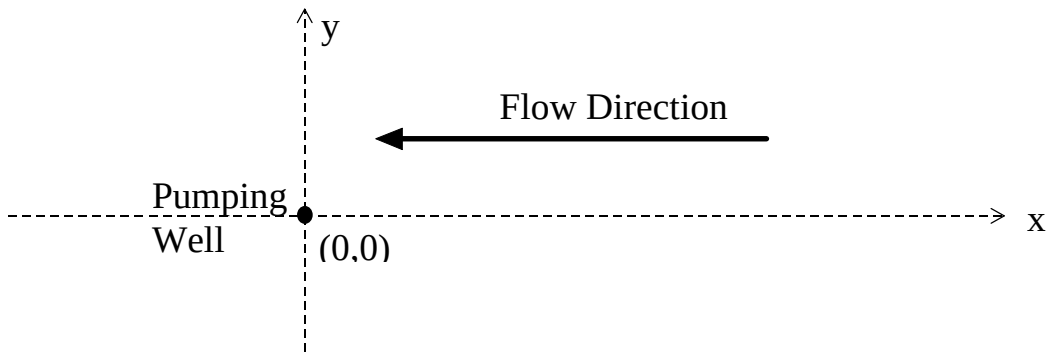
$h(x,y) = h_0 + s_{pw}(x,y) + s_{rg}(x,y)$

Plot head field, draw equipotential lines and flow lines to determine capture zone.

Can expand the above procedure (add more *s* terms) to include:
- Multiple pumping wells
- Image pumping wells to create no-flow boundary
- Image injection wells to create constant head boundary

Alternate procedure for capture zone of well in uniform regional flow field:

Align coordiate system so that regional flow is in x direction:



Determine "stagnation point," or location downgradient of well where net gradient = 0.

$$q_{rg} = -K\frac{\partial h}{\partial x} = -Ki \qquad q_{pw} = \frac{Q}{A} = \frac{Q}{2\pi r_s h} \approx \frac{Q}{2\pi r_s m}$$

Unconfined Aquifer

Confined Aquifer

Stagnation point occurs where $q_{rg} = -q_{pw}$

where: q_{rg} = flux due to regional gradient
i = regional gradient
q_{pw} = flux due to pumping well
Q = pump discharge rate
$r_s = x_s$ = stagnation point
m= average saturated thickness of aquifer

Alternate procedure for single well capture zone (continued):

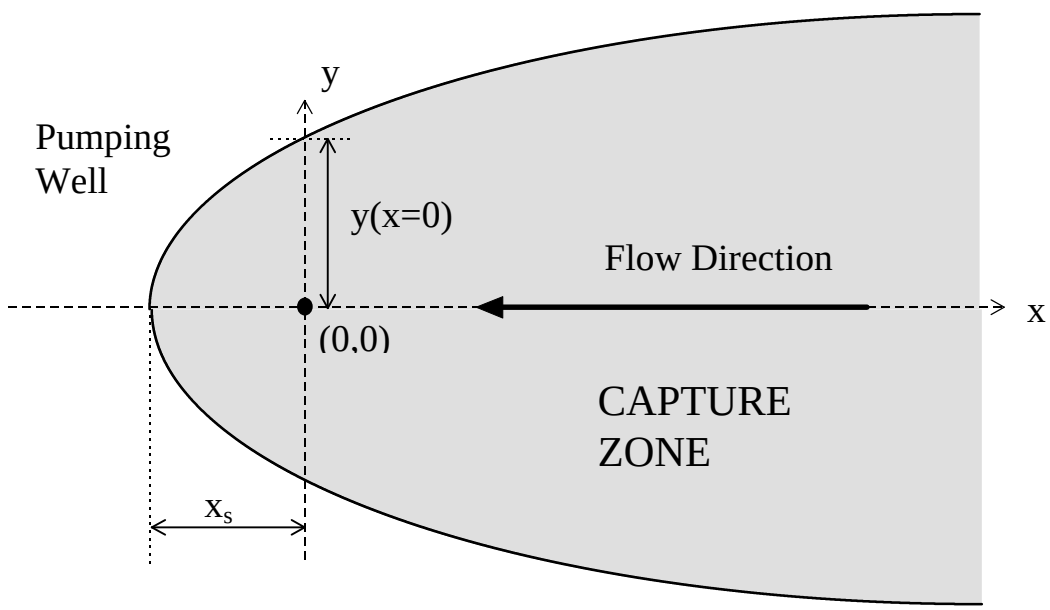
Solve for x_s:

$$x_s = \frac{Q}{2\pi m K i} \qquad \text{note: } x_s < R$$

To describe the rest of the capture zone:

$$\frac{y}{x} = \pm \tan\left(\frac{2\pi K m i}{Q} y\right) \qquad \text{note: } y(x=0) = \pm \frac{\pi x_s}{2}$$

$$\text{and: } y(x=\infty) = \pm \frac{Q}{2Kmi}$$



Alternate procedure for single well capture zone (continued):

y	x
0	-30.49
1	-30.46
10	-29.37
30	-19.93
40	-10.57
47.9	0
60	25.2
70	62.1
80	140.7

Note: signs jump around, beware!