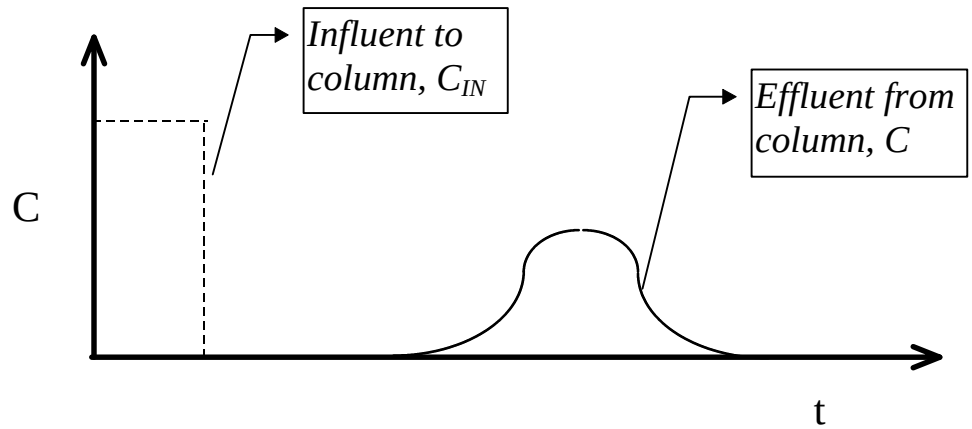
MOMENT ANALYSIS:

- Laboratory column studies and field site investigations give us distributions of contaminant concentration in time and space.
- Can we use this data to determine solute transport parameters?

Temporal moments: Moments based on concentration histories of a solute at a given location (*a.k.a. breakthrough curve*). (e.g.: plume moving past a well; column effluent)

Data:



Absolute temporal moments: Units:

Zero moment: $M_{0,t} = \int_0^\infty C\,dt$ [mass•t/L^3 (water)]

First moment: $M_{1,t} = \int_0^\infty C\,t\,dt$ [mass•t^2/L^3 (water)]

Second moment: $M_{2,t} = \int_0^\infty C\,t^2\,dt$ [mass•t^3/L^3 (water)]

Normalized Moments: Units:

First Moment: $\mu'_{1,t} = \dfrac{M_{1,t}}{M_{0,t}}$ [t]

Second Moment: $\mu'_{2,t} = \dfrac{M_{2,t}}{M_{0,t}}$ [t^2]

Central Moments: Units:

Second Moment: $\mu_{2,t} = \mu'_{2,t} - \left(\mu'_{1,t}\right)^2$ [t^2]

But what does it mean?

For a 1-D system experiencing local equilibrium (i.e., a laboratory column):

- Zero Absolute Moment: $\rightarrow$ *mass*

Total mass of solute = $M_{0,t} \bullet Q$ (Q = flow rate)

Can use information to check for mass balance.

- First Normalized Moment: $\rightarrow$ *travel time ("R")*

$\mu'_{1,t}$ = time when "half" the mass in system has passed the observation location (actually, center of mass of breakthrough curve).

If the solute is not retarded (R = 1):

D = eff. Dispersion coef.

$$\mu'_{1,t} = \frac{L}{v} + \frac{2D}{v^2}$$

v = eff. gw velocity

L = column length

If the solute is retarded (R > 1):

$$\mu'_{1,t} = R\left[\frac{L}{v} + \frac{2D}{v^2}\right]$$

<u>Second Central Moment</u>: →*Dispersion coefficient* (R = 1)

$$\mu_{2,t} = \frac{2DL}{v^3} - \frac{8D^2}{v^4}$$

Assumes dirac load, ±¥ column

Do experiments with sorbing and nonsorbing solutes
Analyze effluent data using moment analysis to:

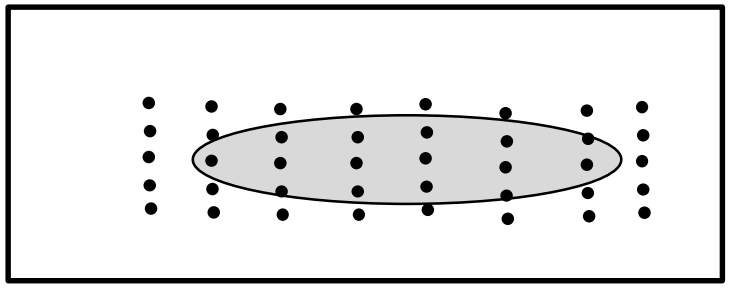
- Make conclusions regarding mass balance
- Estimate retardation factor
- Estimate dispersion coefficient

Questions:

Why is the mean travel time dependent on dispersion coefficient?

How does the 2nd central moment change if R > 1?

Spatial Moment Analysis:



Absolute moments (x direction, 1-D): Units:

Zero moment: $M_{0,x} = \int_{-\infty}^{\infty} C dx$ [mass•L_x/L_w^3]

First moment: $M_{1,x} = \int_{-\infty}^{\infty} C\, x\, dx$ [mass•L_x^2/L_w^3]

Second moment: $M_{2,x} = \int_{-\infty}^{\infty} C\, x^2\, dx$ [mass•L_x^3/L_w^3]

Absolute moments (x direction, 3-D):

Zero moment: $M_{0,0,0} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C\, dx\, dy\, dz$

First moment: $M_{1,0,0} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xC\, dx\, dy\, dz$

Second moment: $M_{2,0,0} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2C\, dx\, dy\, dz$

Units: $M_{0,0,0}$: [$M\, L_m^3/ L_w^3$] $M_{1,0,0}$: [$M\, L_m^4/ L_w^3$]
$M_{2,0,0}$: [$M\, L_m^5/ L_w^3$]

Normalized Spatial Moments:

First Moment: $\mu'_{1,x} = \dfrac{M_{1,x}}{M_{0,x}}$ [L_x]

Second Moment: $\mu'_{2,x} = \dfrac{M_{2,x}}{M_{0,x}}$ [L_x^2]

Central Moments: Units:

Second Moment: $\mu_{2,x} = \mu'_{2,x} - \left(\mu'_{1,x}\right)^2$ [L_x^2]

How can we use spatial moments?

- Mass of plume in aqueous phase, $M_w = M_{1,x}\,\theta_w$
- Location of center of mass of plume = $\mu'_{1,x}$
- Spread of plume = $\mu_{2,x}$

If you have spatial data collected at "snap shots" in time (synoptic data).....

$$\frac{\partial \mu'_{1,x}}{\partial t} = v_{effective,x} = \frac{v_w}{R}$$ *solute velocity in x direction*

$$\frac{1}{2}\frac{\partial \mu_{2,x}}{\partial t} = D_{effective,x} \approx \frac{D_x}{R}$$ *effective dispersion--all sources (not just hydrodynamic)*

Reference: Goltz and Roberts, WRR, 23:1575-1585, 1987